Table 3. Local Nusselt number; $Pr = 0.7$.

θ	Gr 0.01	0.05	0.10	0.20	0.50	1.00
0	1.0094	0.9981	0.9932	0.9854	0.9825	0.9770
20	1.0103	1.0018	0.9991	0.9942	0.9963	0.9958
40	1.0129	1.0123	1.0158	1.0191	1.0348	1.0480
60	1.0167	1.0278	1.0403	1.0554	1.0899	1.1222
80	1.0214	1.0460	1.0686	1.0962	1.1514	1.2036
100	1.0263	1.0645	1.0965	1.1359	1.2093	1.2785
120	1.0308	1.0809	1.1207	1.1695	1.2567	1.3381
140	1.0345	1.0937	1.1391	1.1943	1.2903	1.3788
160	1.0368	1.1017	1.1505	1.2091	1.3098	1.4016
180	1.0376	1.1044	1.1543	1.2141	1.3161	1.4088
$\overline{Nu}$	2.0474	1.1080	2.1596	2.2222	2.3412	2.4520
$\overline{Nu}$[8]		2.09				2.39

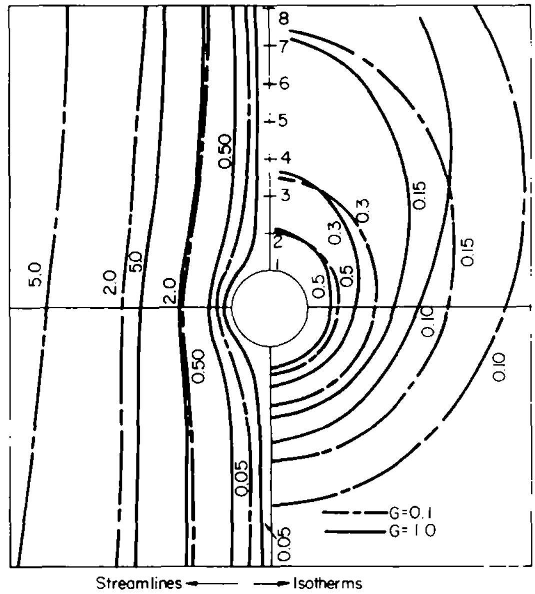

FIG. 1. Streamlines and isotherms for $Gr = 0.1$ and 1.0; $Pr = 0.7$.

fluctuations in the values of Nu for $Gr = 1$, persists for the third truncation. We believe that the fourth truncation should be calculated for $Gr = 1$. From this we conclude that for $Gr < 1$, results for the outer sphere being at a distance of $r_\infty = 60$ may be approximated for a single sphere in an infinite medium. As the Grashof number increases, the rate of heat transfer decreases at $\theta = 0$ and increases near $\theta = \pi$. The increase at $\theta = \pi$ is much more than the decrease at $\theta = 0$. The local Nusselt number for various values of Grashof number are given in Table 3. Table 3 also contains the values of mean Nusselt number calculated by the present method and those of Geoola and Cornish [8] for $Gr = 0.05$ and 1. Comparison is satisfactory. We infer from Fig. 1 that for small values of Gr in the range 0.01–0.1, isotherms are almost concentric spheres and the problem is conduction dominated. For $Gr = 1$ isotherms are nearer to one another at $\theta = \pi$ and tend to become apart near $\theta = 0$. This shows the tendency of plume formation near $\theta = 0$.

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A SECOND LAW ANALYSIS OF THE CONCENTRIC TUBE HEAT EXCHANGER: OPTIMISATION OF WALL CONDUCTIVITY

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NOMENCLATURE

C,	minimum capacity rate $(\dot{m}c_p)_1$;
C_1, C_2,	capacity rate $(\dot{m}c_p)$ of smaller and larger flow rates, respectively;
D,	diameter of inner tube;
h,	heat transfer coefficient;
i,	overall inefficiency;
i_r,	inefficiency ignoring axial conductions but considering the thermal resistance of the partition wall;
I, I_1, I_2,	integrals defined in the text;

k, thermal conductivity;
L, length of the heat exchanger;
N_S, number of entropy production units,* $\dot{S}/C$;
N_{tu}, number of heat transfer units;
Pe, Peclet number;
$\dot{q}$, heat flow rate;
R, inlet temperature ratio, T_2^i/T_1^i;
$\dot{S}$, rate of entropy generation;
t, thickness of partition wall;
T, temperature;
U, overall heat transfer coefficient, $\left[\frac{1}{h_1}+\frac{1}{h_2}+\frac{t}{k}\right]^{-1}$;
x, length coordinate;
X, non-dimensionalised length, x/L.

Greek symbols
ν, capacity rate ratio, C_{min}/C_{max};
ν', capacity rate ratio, C_{tube}/C_{shell};
λ, axial conduction parameter, $k\pi Dt/CL$.

Subscripts
f, fluid;
F, friction;
m, mean;
am, arithmetic mean;
gm, geometric mean;
r, lateral direction;
x, axial direction;
opt, optimum;
1, fluid with smaller capacity rate;
2, fluid with larger capacity rate.

Superscript
i, inlet.

INTRODUCTION

COUNTERFLOW gas-to-gas heat exchangers are extensively used in cryogenic equipment. The design of such heat exchangers has been adequately discussed in many thermal engineering texts [1, 2]. A counterflow heat exchanger, essentially, is made up of two passages, separated by a conducting wall, through which the two fluids flow in opposite direction. A simple unit consists of two concentric tubes, the inner tube providing the heat transfer surfaces. It is, therefore, desirable that the inner tube offers the minimum resistance to heat flow across it. On this argument one would be inclined to provide the material of highest thermal conductivity to the inner tube. On the other hand, high thermal conductivity of the wall enhances axial heat conduction from the hot to the cold end, resulting in reduced effectiveness. Many clever techniques and novel configurations have been devised to achieve large radial heat flow with small axial conduction. However, all the designs are complex, difficult and expensive to manufacture and may develop leaks after prolonged operation. The simple shell-and-tube and concentric-tube configurations are still used for many applications, using a variety of materials, from copper and aluminium to plastic [3]. The optimum material must be chosen considering the fluid properties and flow parameters.

Many thermal processes are now being analysed using the second law of thermodynamics [4]. The application of the second law to the analysis of heat exchangers has been formulated by McClintock [5], Bejan [6, 7], and Sarangi and Chowdhury [8]. Bejan has suggested a design procedure based on the irreversibility analysis in which a combination of heat transfer efficiency and pumping power expended are optimised. However, he has not taken into account other sources of thermal irreversibility, such as axial conduction.

In this article we investigate the effect of axial heat conduction on the performance of concentric tube counterflow heat exchangers and derive an expression for the optimum thermal conductivity of the partition wall. Chowdhury and Sarangi [9] have studied the special case of balanced-flow ($C_{min}/C_{max} = 1$) heat exchanger and have derived an expression for the optimum thermal conductivity as

$$\frac{k_{opt}}{k_f} = \frac{Pe}{4}. \tag{1}$$

Under unbalanced flow ($C_{min}/C_{max} < 1$) condition, k_{opt} also depends upon the capacity rate ratio C_{min}/C_{max}.

The following assumptions may be justified for many counterflow heat exchangers in cryogenic engineering:

(a) The overall N_{tu} is large.
(b) The thickness to diameter ratio of the tube is small.†
(c) Both ends of the heat exchanger are adiabatic.
(d) There is no heat transfer to the environment.

The rate of entropy generation may be expressed as the sum of three terms:

(1) due to heat transfer across finite temperature difference,
(2) due to axial conduction of heat, and
(3) due to frictional pressure drop.

Hence,

$$N_S = (N_{S,r}+N_{S,x})+N_{S,F}. \tag{2}$$

The frictional entropy generation is independent of the thermal conductivity of the wall and also does not affect the thermal effectiveness of the heat exchanger. Hence it will be treated as constant throughout this paper; $N_{S,r}$ and $N_{S,x}$ on the other hand, are coupled through the temperature profiles in the heat exchanger, which are not known *a priori*. However, it may be assumed, without introducing significant error, that the entropy generations due to lateral and axial heat transfer are independent of each other.

ENTROPY GENERATION BY LATERAL HEAT TRANSFER

Consider an infinitesimal length δx of a counterflow heat exchanger [Fig. 1(b)]. The heat transfer and conservation of energy equations may be written as

$$\delta\dot{q}_r = U\Delta T\pi D\delta x \tag{3}$$

and

$$\delta\dot{q}_r = C\delta T_1 = C\frac{dT_1}{dx}\delta x. \tag{4}$$

The entropy generation rate over the infinitesimal length δx may then be expressed as

$$\delta N_{S,r} = \frac{\delta\dot{q}_r}{C}\left(\frac{1}{T_1}-\frac{1}{T_2}\right) = \frac{\delta\dot{q}_r}{C}\frac{\Delta T}{(T_{gm})^2} = \frac{C\delta x}{\pi DU}\left(\frac{1}{T_{gm}}\frac{dT_1}{dx}\right)^2. \tag{5}$$

$$N_{S,r} = \frac{C}{\pi DU}\int_0^L\left(\frac{1}{T_{gm}}\frac{dT_1}{dx}\right)^2 dx = \frac{C}{\pi DUL}\int_0^1\left(\frac{1}{T_{gm}}\frac{dT_1}{dX}\right)^2 dX = I_1/N_{tu} \tag{6}$$

* In ref. [7] N_S is defined as $\dot{S}/C_{max}$ instead of $\dot{S}/C_{min}$ as in the paper.

† The wall thickness is determined by structural requirements and hoof stress resulting from the pressure difference between the two fluid streams.

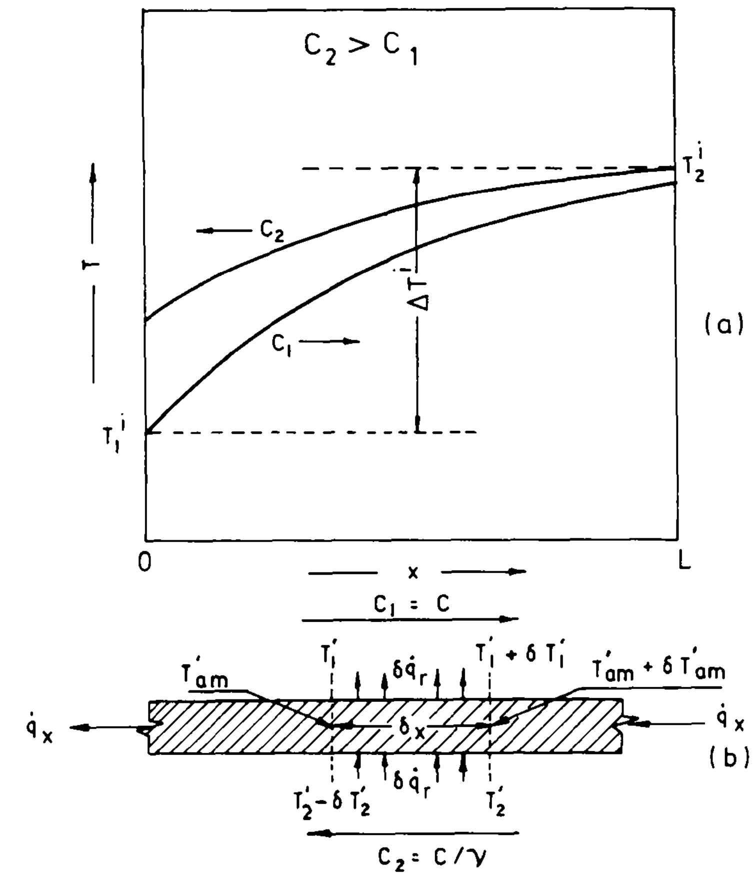

FIG. 1. Temperature profiles in the heat exchanger.

where X is the non-dimensional length x/L and I_1 is the integral

$$\int_0^1 \left(\frac{1}{T_{gm}}\frac{dT_1}{dX}\right)^2 dX.$$

ENTROPY GENERATION BY AXIAL CONDUCTION

$$\delta N_{S,x} = \frac{\dot{q}_x}{C}\left(\frac{1}{T_{am}} - \frac{1}{T_{am}+\delta T_{am}}\right) \simeq \frac{\dot{q}_x}{C}\frac{\delta T_{am}}{(T_{am})^2}. \quad (7)$$

Using the relations,

$$\dot{q}_x = k\pi Dt\frac{dT_{am}}{dx}$$

and

$$\delta T_{am} = (\delta T_1 + \delta T_2)/2 = \left(\frac{1+v}{2}\right)\delta T_1,$$

equation (7) may be expressed as

$$\delta N_{S,x} = \left(\frac{1+v}{2}\right)^2 \frac{k\pi Dt}{C}\left(\frac{1}{T_{am}}\frac{dT_1}{dx}\right)^2 dx$$

and

$$N_{S,x} = \left(\frac{1+v}{2}\right)^2 \frac{k\pi Dt}{C}\int_0^L \left(\frac{1}{T_{am}}\frac{dT_1}{dx}\right)^2 dx = \left(\frac{1+v}{2}\right)^2 \lambda I_2 \quad (8)$$

where λ is the axial conduction parameter $k\pi Dt/CL$ and I_2 is the integral

$$\int_0^1 \left(\frac{1}{T_{am}}\frac{dT_1}{dX}\right)^2 dX.$$

When the overall N_{tu} is large, T_2/T_1 is close to unity and

$$T_{gm} \simeq T_{am} = T_m.$$

Hence,

$$I_1 = I_2 = I = \int_0^1 \left(\frac{1}{T_m}\frac{dT_1}{dX}\right)^2 dX.$$

The integral I is a function of the temperature profiles in the heat exchanger and, to first approximation, may be considered independent of the thermal conductivity of the wall.

The total entropy generation rate may, then, be expressed [from equations (2), (6) and (8)] as

$$N_S = \left[\frac{1}{N_{tu}} + \left(\frac{1+v}{2}\right)^2 \lambda\right] I + N_{S,F}. \quad (9)$$

For optimum performance of the heat exchanger the total number of entropy production units N_S is the minimum and

$$\frac{dN_S}{dk} = 0.$$

Since both I and $N_{S,F}$ are independent of k,

$$\frac{d}{dk}\left[\frac{1}{N_{tu}} + \left(\frac{1+v}{2}\right)^2 \lambda\right] = 0. \quad (10)$$

Using the expressions for N_{tu} and λ, equation (10) yields

$$k_{opt} = \frac{2}{1+v}\frac{C}{\pi D} = \frac{2C_1C_2}{\pi D(C_1+C_2)} \tag{11}$$

which may be termed as

$$\lambda_{opt} = \frac{2}{1+v}\frac{t}{L}. \tag{11a}$$

Equation (11) may be expressed in dimensionless form as

$$\frac{k_{opt}}{k_f} = \frac{2}{1+v'}\frac{Pe}{4} \tag{11b}$$

where k_f and Pe are for the fluid inside the tube and $v' = C_{tube}/C_{shell}$. For the case of balanced flow, the expression reduces to that of ref. [9] by substituting $v' = 1$.

Substituting the expressions (i)

$$\lambda_{opt} = \frac{2}{1+v}\frac{t}{L}$$

and, (ii)

$$\frac{1}{N_{tu,opt}} = \frac{1}{N_{tu,1}} + \frac{v}{N_{tu,2}} + \frac{1+v}{2}\frac{t}{L}$$

into equation (9), the total entropy generation rate under optimum condition may be expressed as

$$N_{S,opt} = \left[\frac{1}{N_{tu,1}} + \frac{v}{N_{tu,2}} + (1+v)\frac{t}{L}\right]I + N_{S,F}. \tag{12}$$

CONCLUSIONS

It has been shown that the optimum thermal conductivity of the partition wall in a concentric tube heat exchanger is a function of the fluid properties, flow parameters and the capacity rate ratio. It is not always advisable to provide material of high thermal conductivity. At high flow velocities copper and aluminium tubes are preferable whereas at low velocities stainless steel and even plastics provide more suitable alternatives.

Kroeger [10] has considered the effect of axial conduction in heat exchanger performance, but, has not explicitly taken into account the effect of lateral resistance. Consequently the question of an optimum thermal conductivity of the partition wall has not been investigated before.

The above analysis may also be applied with suitable modification to shell-and-tube and plate fin heat exchangers and exchangers with longitudinal fins. A shell and tube exchanger may be considered equivalent to a set of concentric tube exchangers operating in parallel.

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AN ANALYSIS OF SUBSTRATE HEAT LOSSES IN STEFAN'S PROBLEM WITH A CONSTANT FLUX

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NOMENCLATURE

a, b, coefficients in the liquid layer temperature profile [$K\ m^{-1}$, $K\ m^{-2}$];
F, constant source per unit area at the substrate interface [$W\ m^{-2}$];
G_i, dimensionless flux from the substrate interface into the liquid-melting solid layer i = l or into the substrate i = s, H_i/F;
$G_{s,\infty}$, asymptotic dimensionless heat flux into the substrate;
H_i, transient heat flux from the substrate interface into